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Calculation method research on the flexural capacity of PSRC beam

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ABSTRACT: This experiment is carried out for the purpose of finding out a proper method that can be used to calculate the flexural capacity of normal section of PSRC beam. According to the test results of 6 experimental beams and two of China's prevailing occupation standard on this field, a new method is put forward, which is specially used to calculate the flexural capacity of this beam. By using comparison analyses, the following is clear: (a) *Simple superposition method* is only suited for special circumstance that is the steel must be located symmetry along the height of the cross-section. However, the error is notable while be used to calculate SRC (steel reinforced concrete) beam. (b) *Coordinate analytical method* based on plane cross-section assumption has better applicability and calculation accuracy on both PSRC beam and SRC beam.

1 PREFACE

Numerous researches on Steel Reinforced Concrete (SRC) structures have been done world widely, among which some of them were completed in China. As for the calculation theory of SRC structure, it is now basically a well-developed system, with three major calculation theories in use world widely, as follows:

a. The *stiffness discounted method* based on steel structure design, with a considering of rigidity reducing caused by outer concrete, is mainly used in the United States.
b. The *ultimate state method* based on concrete structure design is mainly used by the former Soviet Union. According to this method, the flexural capacity of SRC members is calculated analogous to reinforced concrete, but considering the influence of stress distribution of steel ribs. This method is also adopted in the Steel Reinforced Concrete Structure Technical Regulations of China.
c. The *additive method*, mainly used in Japan, is to overlap the flexural capacities of concrete and steel together as the final flexural capacities of SRC members. This method can also be seen in Steel Reinforced Concrete Design Codes of China.

As for flexural capacity of Prestressed Steel Reinforced Concrete (PSRC) beams, few researches have been done world widely; additionally, the current two regulations of China have different viewpoints on the collaboration of concrete and steel. For reasons above, experiment is carried out. From the test results of 6 PSRC beams, and considering the influence of prestress, the *simple additive method* and *coordinate analytical method* based on the current two regulations of China were put forward to calculate the flexural capacity of such beam. In addition, evaluations upon rationality and applicability of these two methods were completed after comparing the results of experiment and theoretical calculation.

2 SIMPLE ADDITIVE METHOD

Now borrowing ideas from the ninth reference, regarding that the PSRC beam which is consist of steel ribs (S) and prestressed reinforced concrete (PRC), thus the flexural capacity of PSRC beam can be acquired by adding the two part's flexural capacities together. The calculation formula is given by

$$M \le M_{bu}^{prc} + M_{by}^{ss} \tag{1}$$

where M = design value of bending moment at section; M_{by}^{ss} = flexural capacity of steel ribs; M_{bu}^{prc} = flexural capacity of PRC department.

If the earthquake action dose not counts in, the flexural capacity of the steel is calculated by

$$M_{by}^{ss} = \gamma_s f_{ss} W_{ss} \tag{2}$$

where W_{ss} = elastic resistance moment of cross section of steel; γ_s = plastic coefficient of cross section, which is used to lower the calculation error caused by the difference between pure bending model and real eccentrically tension situation of steel section (for I-shape steel $\gamma_s = 1.05$); f_{ss} = design strength of steel ribs under tension, compression, or bending

The flexural capacity of PRC department is represented by the equation

$$M_{bu}^{prc} = \gamma h_{b0} \cdot (A_s f_{sy} + A_p f_p) \tag{3}$$

where A_s = area of re-bar under tension; A_P = area of prestressed reinforcement; f_{sy} = design value of tensile strength of re-bar; f_P = design value of tensile strength of prestressed reinforcement; h_{b0} = the distance from the action spot of resultant tensile force of reinforcement (including re-bar and prestressed reinforcement) to the outer boundary of compressive region; γh_{b0} = the distance from the action spot of resultant tensile force to the action spot of resultant compressive force; γ = coefficient of internal force arm. According to document 7, $\gamma = 0.875$.

SRC beams can be considered as PSRC beams with a prestressed degree equal 0; hence the *simple additive method* is suit for SRC beams also. That is to say, with a universal applicability.

3 COORDINATE ANALYTICAL METHOD

3.1 *Failure morphology*

Numerous researches shows that, no matter SRC beams or PSRC beams as long as the cross-section well designed, and enough shear keys being putted between the steel's compressive flange and concrete, the over reinforced failure of PSRC beam seldom occurs while at ultimate state of failure. The failure morphology is similar to a well-designed RC beam that is, the failure occurs by yielding of tensile reinforcement (including re-bar, prestressed reinforcement, flange and partial web of steel ribs), not by crushing of concrete.

3.2 *Fundamental assumption*

For rectangular PSRC beam with solid-web steel encased, the following hypotheses must be obeyed when calculate the flexural capability of this beam:

a. Each material on the cross section obeys the plane cross-section assumption. Data in the sixth document demonstrate that, before the bottom flange of steel yielding, this assumption applies well. Even if the bottom flange of steel yielded, for little growth of flexural capability generated after that, hence the plane cross-section assumption can also be used in analysis, and without a significant error.
b. The ultimate compression strain of concrete at compressive region is given by $\varepsilon_{cu} = 0.003$.
c. At ultimate state, the distribution of compressive stress at compressive region is assumed as a rectangular distribution. The equivalent height of this compressive region can be expressed by β_1 times the depth of compression zone obtained by the plane cross-section assumption. The corresponding maximum compressive stress is equal to the compressive strength f_c.
d. For rebar and steel rib, ideal elastoplastic constitutive relation of stress-strain (σ–ε) is adopted. According to full plastic assumption, the stress distribution of the web of steel rib can be simplified to two rectangular stress distribution models, one in tensile, the other in compressive. When at ultimate state, because of the thin thickness and the short force arm of the web, the contribution of it to the total flexural capability is small, thus the tensile region of the web can be considered as a full yield region.
e. Tensile strength (f_{py}) of prestressed reinforcement being arrived while at ultimate state.
f. The tensile strength of concrete is ignored.
g. Local buckling of steel rib does not occur.
h. In terms of the assumption above, the distribution of strains, actual stresses, and simplified stresses in concrete and steel rib over the depth of the section at ultimate state is as shown in Fig. 1.

3.3 *The flexural capacity of normal section of solid-web PSRC beams*

3.3.1 *Judgment of stress state at ultimate state*

At ultimate state the relative depth of compression zone ξ is given by

$$\xi = x/h_0 \tag{4}$$

where x = height of equivalent rectangular stress diagram of compressive region of concrete; h_0 = the distance from the outer boundary of compressive region to the action spot of resultant tensile force. These tensile forces including the force in re-bar, prestressed reinforcement, and tensile flange of steel rib, however the force in tensile web is ignored for little contribution to the total flexural capacity.

To determine the exact stress status of normal section at ultimate state, the critical relative depth ξ_i of compression zone of the following several situations need to be understood first:

a. At this condition, the strain of top steel flange is exactly equal to the tensile yielding strain when the concrete strain in outer compression fiber simultaneously reaches the crushing strain ε_{cu}. Thus, the critical relative depth ξ_1 of compression zone can be determined, as follow:

$$\frac{\xi_1 \frac{h_0}{\beta_1}}{a'_a - \xi_1 \frac{h_0}{\beta_1}} = \frac{\varepsilon_{cu}}{\varepsilon_{ay}} \Rightarrow \xi_1 = \frac{(a'_a - \xi_1 \frac{h_0}{\beta_1}) \cdot \varepsilon_{cu}}{h_0 \cdot \varepsilon_{ay}} \cdot \beta_1 \tag{5}$$

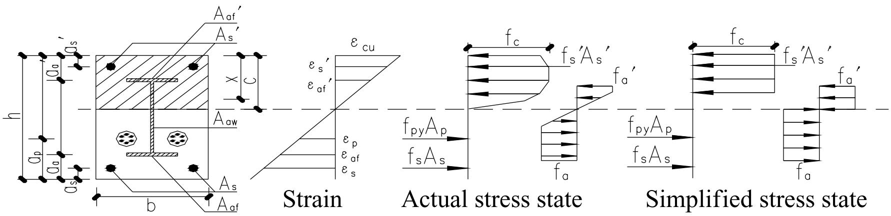

Figure 1. Stress and strain of normal section at ultimate state.

where β_1 = the ratio of height of equivalent rectangular stress block to the depth of neutral axis. If the strength grade of concrete is lower than C50, in this case $\beta_1 = 0.8$; If the strength grade of concrete equal C80, then $\beta_1 = 0.74$; Else if the strength grade is just between C50 and C80, the value of β_1 can be calculated by linear interpolation method.

b. The stress of top steel flange is exactly equal to zero when the concrete strain in outer compression fiber simultaneously reaches the crushing strain ε_{cu}. According to plane cross-section assumption, the critical relative depth ξ_2 of compression zone can be derived from

$$\xi_2 h_0/\beta_1 = a'_a \Rightarrow \xi_2 = a'_a/h_0 \cdot \beta_1 \tag{6}$$

c. At this condition, the strain of top steel flange is exactly equal to the compressive yielding strain when the concrete strain in outer compression fiber simultaneously reaches the crushing strain ε_{cu}. Thus, according to plane cross-section assumption, the critical relative depth ξ_3 of compression zone can be derived from

$$\frac{\xi_3 h_0/\beta_1}{\xi_3 h_0/\beta_{1\,0} - a'_a} = \frac{\varepsilon_{cu}}{\varepsilon'_{ay}} \Rightarrow \xi_3 = \frac{a'_a \varepsilon_{cu}}{h_0 \cdot (\varepsilon_{cu} - \varepsilon'_{ay})} \cdot \beta_1 \tag{7}$$

d. At this condition, the strain of bottom steel flange is exactly equal to the tensile yielding strain when the concrete strain in outer compression fiber simultaneously reaches the crushing strain ε_{cu}. Thus, according to plane cross-section assumption, the critical relative depth ξ_4 of compression zone can be derived from

$$\frac{\xi_4 h_0/\beta_1}{h - a_a} = \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{ay}} \Rightarrow \xi_4 = \frac{\varepsilon_{cu} \cdot (h - a_a)}{h_0 \cdot (\varepsilon_{cu} + \varepsilon_{ay})} \cdot \beta_1 \tag{8}$$

e. At this condition, the strain of bottom longitudinal steel bar is exactly equal to the tensile yielding strain when the concrete strain in outer compression fiber simultaneously reaches the crushing strain ε_{cu}. Thus, according to plane cross-section assumption, the critical relative depth ξ_5 of compression zone can be derived from

$$\frac{\xi_5 h_0/\beta_1}{h - a_s} = \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{sy}} \Rightarrow \xi_5 = \frac{\varepsilon_{cu} \cdot (h - a_s)}{h_0 \cdot (\varepsilon_{cu} + \varepsilon_{sy})} \cdot \beta_1 \tag{9}$$

f. At this condition, the strain of prestressed reinforcement is exactly equal to the tensile yielding strain when the concrete strain in outer compression fiber simultaneously reaches the crushing strain ε_{cu}. Thus, according to plane cross-section assumption, the critical relative depth ξ_6 of compression zone can be derived from

$$\frac{\xi_6 h_0/\beta_1}{h - a_p} = \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{p1}} \Rightarrow \xi_6 = \frac{\varepsilon_{cu} \cdot (h - a_p)}{h_0 \cdot (\varepsilon_{cu} + \varepsilon_{p1})} \cdot \beta_1 \tag{10}$$

where ε'_{ay}, ε_{ay}, ε_{sy} = they are strains of the top compressive flange of steel, the bottom tensile flange of steel, and the bottom longitudinal tensile re-bar. ε_{p1} = strain increment of prestressed reinforcement from the state of none stress exist in concrete near the action spot of prestressing tendon, to the state of yielding of the this tendon.

By sorting these ξ_i above, and comparing them with the actual ξ, the real stress distribution of each component at ultimate state can be known clearly.

3.3.2 *Critical depth of relative compression zone (ξ_b)*

The constituents of PSRC members is more complex than usual RC members; also the position and relative stress value of tensile re-bar, steel rib, and prestressed reinforcement are uncertain. It is concluded that the way to ascertain ξ_b only by one material is inapplicable. As known to all, the concrete in compressive region reaches its ultimate compressive strain ε_{cu} dose not means all of the prestressed reinforcement, tensile re-bar, and the bottom flange of steel yield at the same time. By analysis each ξ_i above, it is cleared that, under the ξ_1 and ξ_2 situation, all of the prestressed reinforcement, tensile re-bar, and the bottom flange of steel yield while the concrete in compressive region reaches its ultimate compressive strain ε_{cu}. Based on the discussion above, the value of ξ_3, ξ_4, ξ_5 or ξ_6 is relative lager than ξ_1 or ξ_2. Therefore, to assure all the tensile reinforcements yielding while the beam is at the critical failure state, ξ_b is given by

$$\xi_b = \text{Min}(\xi_3, \xi_4, \xi_5, \xi_6) \tag{11}$$

The above method to ascertain ξ_b is precise and conservative enough, but the process is more complex. Hence the simplified method based on current specifications and codes of China is put forward. that is to divide the PSRC beam into two parts, the PRC department and the SRC department, and calculate the ξ_b of each part separately, then take the smaller one as the PSRC beam's ξ_b at last. The ξ_b calculated in this method is relatively small, so the calculated flexural capability is more or less smaller than actual flexural capability. According to the third reference, the ξ_b of the PRC department is given by

$$\xi_{pb} = \frac{\beta_1}{1 + \frac{0.002}{\varepsilon_{cu}} + \frac{f_{py} - \sigma_{po}}{E_p \varepsilon_{cu}}} \tag{12}$$

where σ_{po} = stress of prestressed reinforcement at the moment of zero normal stress state of concrete next to the prestressed reinforcement; E_P = elasticity modulus of prestressed reinforcement.

The value of ξ_{pb} can also be taken referring to the Highway Reinforced Concrete and Prestressed Concrete Bridges Design Specification of China, as seen in Table1. If different kinds of re-bar are located at the tensile region, the value of ξ_{pb} equals to the minimum one in Table 1 below.

Referring to the ninth reference, and considering the compatibility of it to the third reference, β_1 is introduced. Then

$$\xi_{sb} = \frac{\beta_1}{1 + \frac{f_{sy} + f_{ay}}{2 \times E_s \varepsilon_{cu}}} \tag{13}$$

where f_{ay} = design value of tensile strength of steel rib; E_S = elasticity modulus of rebars.

3.3.3 *Depth of compression zone of concrete*

At ultimate state, three conditions about the location of neutral axis in cross section are sorted. They are the neutral axis dose not go through the steel, the neutral axis just through the top flange of the steel, and the neutral axis through the web of steel. The condition that the neutral axis through the top flange of the steel is thought as a criterion to distinguish the other two as mentioned above, and the simplified state of stress and strain under this condition is shown in Figure 2.

The actual depth (c) of compression zone of concrete can be determined by the following trial method. First the assumption that the neutral axis just going through the top flange of steel is adopted, that is to say $\xi_{af} = \xi_2$. Then compare of the total tensile force T_{af} and the total compressive force P_{af} at the same cross-section, as follows:

1. $T_{af} = P_{af}$, then $x = \beta_1 c$.
2. $T_{af} > P_{af}$, it is means the actual depth of compression zone of concrete is deeper than the assumed one, then another assumption $\xi = \xi_3$ is adopted, and once again to compare the total tensile force T and the total compressive force P across the section.
   - if $T = P$, in this case $x = \beta_1 c$.
   - if $T < P$, it is means the actual depth of compression zone of concrete is shallower than the assumed one, thus x can be calculated according to plane cross-section assumption and equilibrium condition of the section.
   - else if $T > P$, it is means the actual depth of compression zone of concrete is deeper than the assumed one, or the top flange of steel have already yielded, thus x can be calculated by equilibrium condition of the section.
3. $T_{af} < P_{af}$, it is means the actual depth of compression zone of concrete is too shallow that the actual neutral axis dose not reach the top flange of steel rib, then the assumption $\xi = \xi_1$ is adopted.

Correspondingly, compare the total tensile force T and the total compressive force P across the section.

- if $T = P$, in this case $x = \beta 1c$.
- if $T < P$, it is means the actual depth of compression zone of concrete is shallower than the assumed one, the total cross section of steel have already

Table 1. Relative compressive area depth at limit state.

	Concrete grade	C50 and below	C55/C60	C65/C70	C75/C80
Types of steel reinforcement	HPB235	0.62	0.60	0.58	–
	HRB335	0.56	0.54	0.52	–
	HRB400, RRB400	0.53	0.51	0.49	–
	Strand, steel wire	0.40	0.38	0.36	0.35
	Planished concrete reinforcing bar	0.40	0.38	0.36	–

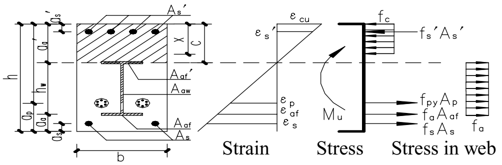

Figure 2. Stress and strain of cross section with neutral axis through top flange of steel.

yielded, and x can be calculated by equilibrium condition of the section.

– else if $T > P$, it is means the actual depth of compression zone of concrete is deeper than the assumed one, the top flange of steel is in tensile state without yielding, and x can be calculated according to plane cross-section assumption and equilibrium condition of the section.

In addition, x must satisfy the $\xi < \xi_b$ condition so as to make sure the ductile fracture appearing while at ultimate state, that is the failure initiates by yielding of rebar, prestressed reinforcement, and the bottom flange of steel yielding at ultimate state. If this condition can not be satisfied, adjustment of the sectional dimension or concrete grade is suggested to meet this requirement.

3.3.4 *Flexural capacity M_u*

For convenience, the height of steel web h_w is replaced by the distance between the center lines of top flange and bottom flange, and the compression area of concrete is calculated without deduction of the area of steel rib. The stress of the top flange can be expressed by σ'_{af} which may be tensile or compressive, yielding or not.

For situation I, the neutral axis just goes through the top flange of steel (see Fig. 2), taking moment about the top flange of steel gives

$$\begin{aligned} M_u = & f_c bx \cdot (a'_a - x/2) + f'_s A'_s \cdot (a'_a - a'_s) \\ & + f_a A_{aw} h_w/2 + f_s A_s \cdot (h - a'_a - a_s) \\ & + f_a A_{af} h_w + f_{py} A_p \cdot (h - a'_a - a_p) \end{aligned} \quad (14)$$

For situation II (see Fig. 3), taking moment about the neutral axis gives

$$\begin{aligned} M_u = & f_c bx \cdot (x/\beta_1 - x/2) + f'_s A'_s \cdot (x/\beta_1 - a'_s) \\ & + \sigma'_{af} A'_{af} \cdot (x/\beta_1 - a'_a) + 0.5\sigma'_{af} t_w \\ & \cdot (x/\beta_1 - a'_a)^2 + 0.5 f_a t_w \cdot (h_w - x/\beta_1 + a'_a)^2 \\ & + f_a A_{af} \cdot (h_w - x/\beta_1 + a'_a) + f_s A_s \\ & \cdot (h - a_s - x/\beta_1) + f_{py} A_p \cdot (h - a_p - x/\beta_1) \end{aligned} \quad (15)$$

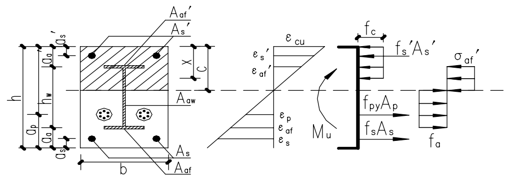

Figure 3. Stress and strain of cross section when neutral axis through the web.

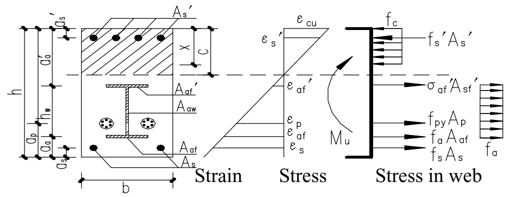

Figure 4. Stress and strain of cross section when neutral axis dose not pass the steel skeleton.

For situation III (see Fig. 4), taking moment about the top flange of steel gives

$$\begin{aligned} M_u = & f_c bx \cdot (a'_a - x/2) + f'_s A'_s \cdot (a'_a - a'_s) \\ & + f_a A_{aw} h_w/2 + f_s A_s \cdot (h - a'_a - a_s) \\ & + f_a A_{af} h_w + f_{py} A_p \cdot (h - a'_a - a_p) \end{aligned} \quad (16)$$

4 COMPARISON BETWEEN THEORETICAL CALCULATION AND TEST RESULTS

In order to understand the force performance of PSRC beams and find out the design method of flexural capability of it, three groups of 6 simple beams were researched. The 6 specimens are consisting of 2 SRC beams and 4 PSRC beams. The prestressed reinforcement is composed of two bunches of prestressed tendon with six 5 mm high-strength steel wire in each bunch. Steel rib adopted is Q235 hot rolled steel H-beams (HN200 ×100 × 5.5 × 8), as shown in Figure 5. Comparison of theoretical calculation and test results is shown in Table 2. According to the data in Table 2 the following is clear: (a) *Simple superposition method* is only suit for special circumstance that is the steel must be located symmetry along the height of the cross-section. However, the error is notable while be used to calculate SRC (steel reinforced concrete) beam. (b) *Coordinate analytical method* based on plane cross-section assumption has better applicability and calculation accuracy on both PSRC beam and SRC beam. It is found that the calculated result using *coordination analytical method* not only close to the measured value, but also conservative enough.

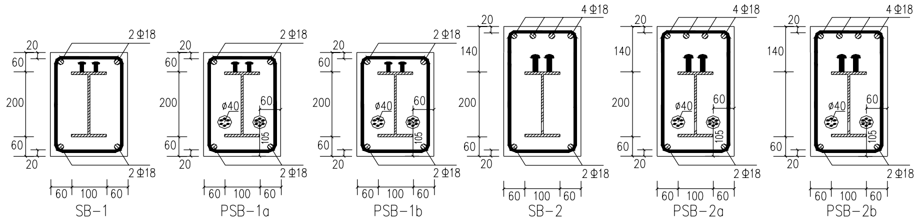

Figure 5. Dimensions and reinforcements of specimens.

Table 2. Test values and theory values of the carrying capacity for the specimens (kn·m).

Beam number	SB-1	PSB-1a	PSB-1b	SB-2	PSB-2a	PSB-4b
Type of test beam	SRC[1]	PSRC[1]	PSRC[1]	SRC[2]	PSRC[2]	PSRC[2]
Degree of prestress	0	0.160	0.173	0	0.160	0.173
Measured value of M_u	151.90	200.90	199.90	240.10	313.60	312.60
(Simple superposition method) M_u^1	112.90 (25.67%)	176.22 (12.28%)	175.58 (12.17%)	125.98 (47.53%)	212.88 (32.12%)	212.02 (32.18%)
(Coordination analytical method) M_u^2	148.54 (2.21%)	198.57 (1.16%)	198.70 (0.6%)	239.20 (0.37%)	300.32 (4.23%)	298.54 (4.50%)

Note: 1) Superscript 1 means that the steel is symmetrical in layout while subscript 2 means unsymmetrical layout of steel. 2) Relative error between computed value and measured value of bending moment is shown in the parentheses.

5 CONCLUSIONS

a. *Simple superposition method* making use of the concept in China's Steel Reinforced Concrete Design Codes is a simple calculation, suit for manual computation with a higher accuracy for PSRC beam than SRC beam. However, it is only suit for special circumstance that is the steel must be located symmetry along the height of the cross-section; furthermore, the result is conservative excessively. Because of these reasons, this method can be used for scheme selection and preliminary design just in terms of its characters.
b. *Coordinate analytical method* with tight logical derivation, combines China's current specifications and codes well. Hence, not only this method can be used for all kinds of arrangement of steel rib, but also the calculated value is close to the experimental result. However the computing process is complex. Therefore, this method is quite suit for computer calculation.
c. Both of the two methods have universal applicability for SRC and PSRC structure. Not only rectangular section but also I-shaped and T-shaped sections can be calculated by the two methods

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