

# Price Targets and Trading Behavior

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Abstract I develop a new model to analyze trading and pricing dynamics in financial markets with informational frictions. My model demonstrates that in a standard framework with rational investors, a simple but natural specification of gradual information diffusion can explain many observed stylized facts about individuals' trading behavior in asset markets. In particular, the model generates trading behavior that spans the axis from "disposition effect" behavior on one extreme, to "momentum chasing" on the other extreme, without recourse to preference-based behavioral explanations. My model rationalizes the empirical findings that trading varies along this disposition-momentum axis according to the characteristics of the underlying asset market, and it generates a variety of other novel empirical predictions.

## Part I Introduction

The tendency of investors to realize their gains and avoid realizing their losses has been documented in a large body of empirical literature. In the seminal paper of this literature, Shleifer and Statman (1985), termed this tendency the disposition effect, and they proposed an explanation based on the Prospect Theory developed by Kahneman and Tversky (1979); both the terminology and the explanation have endured. Using trade-level data from a large discount brokerage, Odean (1998) provided strong empirical evidence for the existence of a disposition effect, and he argued that his results could not be well explained by classical considerations such as portfolio rebalancing, tax-loss selling or belief in mean reversion. As Barberis and Xiong (2009) note in their recent paper, the disposition effect is most commonly explained by preference-based models that use non-standard utility specifications (usually dubbed "loss aversion") derived from Prospect Theory.

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However, despite the widespread acceptance of preference-based explanations of the disposition effect, preference-based theories are problematic on several levels. First, using Odean's (1998) data, Rangelova (2000) finds that the disposition effect is concentrated in large-cap stocks, and that the disposition effect actually reverses for stocks in the bottom quintile of market cap. Moreover, Rangelova shows that her results are robust to individual effects, margin calls, and a variety of other controls (Given this variation in trading behavior, I will use the term disposition behavior to refer to where trading tendencies fall on the disposition effect/anti-disposition effect spectrum; I will reserve the term disposition effect to specifically refer to those instances where investors realize gains more readily than losses). Similarly, Calvet et al. (2009) find that the disposition effect is less pronounced among mutual funds than among individual stocks; while performance chasing may account for part of this reduction, variation in the disposition effect as a function of the identity of the underlying asset is prima facie evidence against preference-based explanations.

Preference-based explanations of the disposition effect are also somewhat problematic on theoretical grounds. For example, Barberis and Xiong (2009) present evidence that popular preference-based models sometimes fail to deliver a disposition effect, and the failures do not bear any obvious relationship to the empirically observed settings in which the disposition effect is not observed or reverses. Even more troubling is the nebulous notion of a "reference point" that plays a central role in prospect-theoretic formulations. In static settings, the convention of setting the reference point equal to initial purchase price is fairly innocuous, but extensions to dynamic models—necessary to explain trading behavior—must impose strong, ad hoc assumptions about how people update their "reference points" over time. Finally, most prospect-theoretic preference-based models must invoke some form of "narrow framing" or "mental accounting" to give loss-aversion any bite (cf. Barberis et al., (2006)), and specifications of "narrow framing" introduce additional degrees of freedom on top of the already hazy "reference point" apparatus.

In her 2000 paper, Rangelova suggested that her results could be explained by a model in which investors had "price targets," which they updated at rates that depended on the characteristics of the underlying stock. In this paper, I formalize this intuition by developing a rational model of price-target setting and updating. I build up on the fundamental insight, introduced by Grossman and Stiglitz (1980) and developed in literature on heterogeneous beliefs in financial markets, that in the presence of market noise, market-clearing prices need not fully reveal all private information. In my model, a single investor (assumed small relative to the aggregate market) observes a private signal about the fundamental value of a stock, but due to the presence of noise traders, this private signal is not immediately incorporated into market prices. The investor uses his initial private signal to form a price-target; thereafter, information about the true fundamental value of the stock diffuses to the public, and the investor uses this information to update his price-target. Since the market price also reacts to the diffusion of information, the model generates predictions about how the investor's trades will move relative to the market price.

Other than assuming that market prices do not fully reveal all private information, which is not particularly unorthodox in the modern finance literature, I make only one non-standard assumption. In the baseline case, the private signal that the single investor observes is “binary” in the sense that it either reveals the true fundamental value of the stock, or it contains no information whatsoever about fundamental value. The model easily extends to the less extreme setting where the signal either reveals a noisy observation of fundamental value, or reveals nothing, but the binary true/false nature of the signal is vital. The binary structure of the signal causes the investor’s Bayesian updates of his beliefs about the fundamental value of the stock to evolve very differently than they would if he had just observed a noisy signal of fundamental value, say of the form  $\text{signal} = F + V + \epsilon$ , with  $\epsilon \sim N(0, s)$ . In order to analyze my model and tease out its empirical implications, I also introduce a new technique for characterizing disposition behavior in theoretical models, and I develop some results that characterize the link between empirically observed average disposition behavior and the disposition behavior that a model predicts for a single investor.

The remainder of this paper is organized as follows: Part I builds up to the workhorse price-target updating model through a succession of simpler models, and discusses empirical implications; Part II examines in detail the theoretical issues associated with the characterization and observation of disposition behavior; Part IV concludes.

## Part I

# Models of Asynchronous Information Arrival and Price Targets

In this part, I present a sequence of models of increasing complexity which help to illuminate the effects of gradual information diffusion on the trading behavior of investors. The first two models have little empirical content in their own right, but they provide a foundation for the subsequent models, which have considerable empirical content.

<sup>1</sup>A slightly more conventional characterization of the signal, in this case, would be to assume that it is generated in two stages. In stage one, a Bernoulli random variable,  $b$ , is drawn. In stage two, if the signal is drawn from the distribution  $N(V + \sigma(b - V), s^2)$  if  $b = 1$ , and the signal is drawn from the distribution  $N(-E[V], s^2)$  if  $b = 0$ , where  $V$  denotes the true fundamental value, and  $E[V]$  denotes the expectation of fundamental value conditioned up on information publicly available at  $t = 0$ .

Investors myopically optimize their portfolios with respect to this utility function in each period. At time  $t$ , investors believe  $V \sim N(\mu, \sigma^2)$ ,

$$U_t = E_t \left[ -\exp \left( -\frac{1}{2} \frac{(V - P_t)^2}{\sigma^2} \right) \right]$$

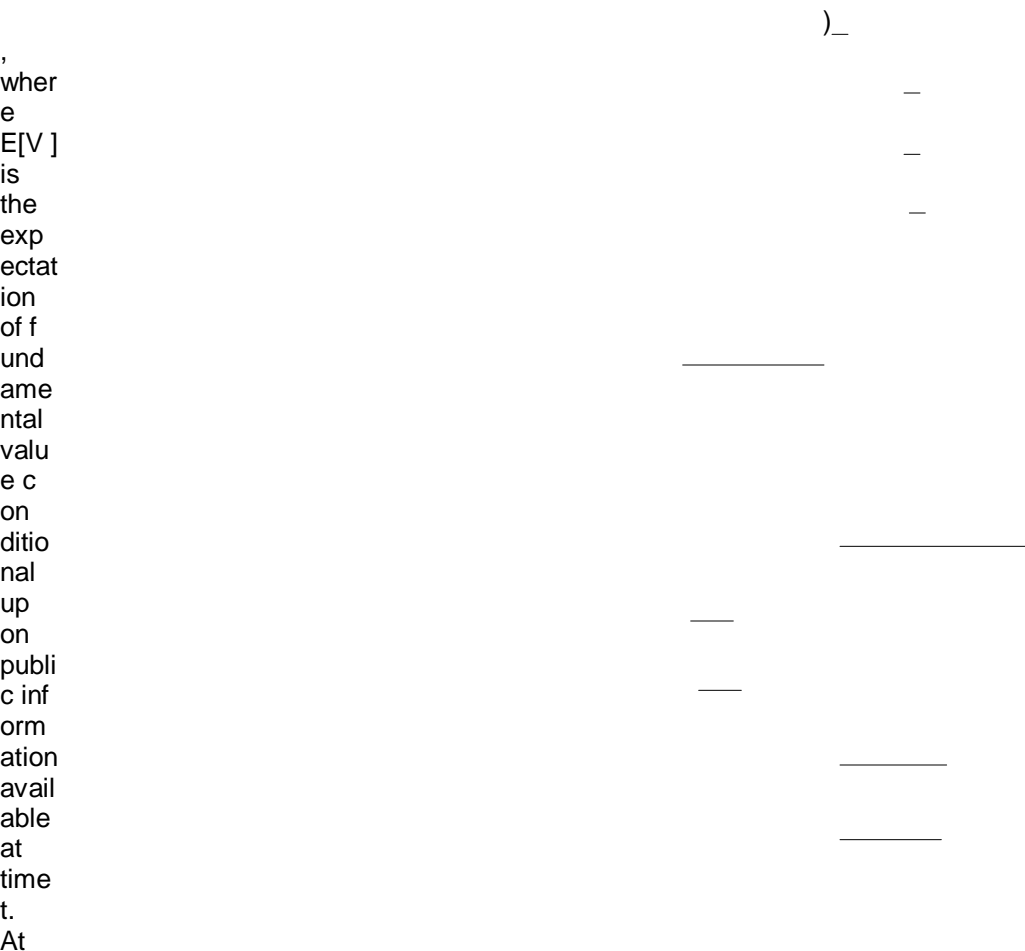
$$\max_x E_t \left[ -\exp \left( -\frac{1}{2} \frac{(V - P_t)^2}{\sigma^2} \right) \right]$$

## 1 One Potentially Informed Trader, One Signal

### 1.1 Market Structure

Time is discrete, indexed by  $\{0, 1, 2, \dots\}$ . There are two assets, a risk-free asset, with a return normalized to zero, available in perfectly elastic supply ("cash"), and a risky asset ("stock"), available in net supply normalized to unity. The fundamental value of the stock is  $V$ .

The market consists of a large number,  $N$ , of homogeneous investors, each with mean-variance utility over the fundamental value of next-period wealth, given by



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$$\begin{aligned} & \frac{1}{2} \text{var} [V - P_t] \\ & \frac{1}{2} \text{var} [V] \\ & \frac{1}{2} \text{var} [V - P_t] \\ & \frac{1}{2} \text{var} [V] \\ & \frac{1}{2} \text{var} [V - P_t] \\ & \frac{1}{2} \text{var} [V] \end{aligned}$$

$$= N x^* t$$

$$\frac{1}{2} \text{var} [V - P_t] = \frac{1}{2} \text{var} [V] - P_t = \gamma S x^* t = E M_t$$

Market-clearing then requires P

$$\begin{aligned}
& \frac{[V] - P}{2\gamma S} = E_t \\
& N P \quad \quad \quad M [V] - P \\
& 1 + 2\gamma S N = E_{M t} [V] \\
& P_t \\
& P_t = N E_{M t} [V] N \\
\Rightarrow x^* t & = \frac{+ \gamma S}{\gamma S^2} E_{M t} [V] N + \\
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\end{aligned}$$

$\tilde{P}_t$  is given by  $\tilde{P}_t = P_t$ . Now, suppose that there are some noise traders active in each period, and they demand (or supply) some random quantity  $d$  inelastically, so that residual supply is  $1 - d$ , and the market price is  $P_t / (1 - d)$ . Assume that  $d \sim U[-c, c]$  for some small positive constant  $c$ . Thus the market price will stay in some small  $c$ -neighborhood of  $E_{M_t}[V] + \gamma S_t$ ; whether we suppose that these tiny, transient deviations from  $E_{M_t}[V] + \gamma S_t$  are non-zero, due to small unmodeled frictions, or instead assume that they are arbitrated away is irrelevant for the purposes of this model. The noise traders are introduced simply to circumvent no-trade results.

## 1.2 Information Structure

$E_{M_t}$  At time 0, the fundamental value of the stock,  $V$ , is not known to investors, and I assume that  $E_{M_0}[V] + \gamma S_0 = P_0 = V$ . For concreteness, I will suppose that  $P_0 < V$ , as this falls naturally out of the risk-compensation framework, but there would be little conceptual difference in supposing  $P_0 > V$ . At the beginning of each date  $t = 1$ , with some fixed probability  $\theta > 0$ , the true fundamental value  $V$  is publicly revealed, if it has not already been revealed in an earlier period (i.e. the public revelation of  $V$  follows a Poisson arrival process). When the fundamental value  $V$  is publicly revealed at time  $\tilde{t}$ , market expectations change so that

$E_{M_{\tilde{t}}}[V] = V$  and consequently  $P_{\tilde{t}} = V$  since there is no longer any risk and hence no risk discount. (We could also assume that  $P_{\tilde{t}} = V + \frac{\theta}{5} (1 - \theta)^{\tilde{t} - t}$  for  $\{t\}_{t=0}^{\tilde{t}}$ )

$t_{t+1}$ , but it is not clear how noise traders would affect the market if when fundamental value is common knowledge. It may be appropriate to allow some form of non-negative noise in the price).

Now, consider a single investor, A. At date 0, investor A receives a signal,  $S$ , which he alone observes. With probability  $t < 1$ ,  $S = V$ , i.e., the signal reveals the fundamental value of the stock. However, with probability  $1 - t$ , the signal is completely uninformative, and has no information about the stock's fundamental value. I will call a signal true if it reveals the fundamental value of the stock, and false if it is instead uninformative. I will refer to A as Potentially Informed Trader or PIT.

PIT knows that if his signal is true, then in each period there is a constant,

independent probability,  $\theta$ , that  $S = V$  will be made public. However, if PIT's signal is false, then there is a zero probability that  $S$  will be made public. This means that every time a period elapses without  $S$  being publicly revealed, PIT can revise his estimate of  $t$ . Specifically, assume that PIT updates his estimates in a Bayesian manner, so that he generates a sequence of estimates according to the following recursive rule:



Note that  $\{t_t\}_{t=0}^T$

$t_{t+1}$  is a strictly decreasing sequence:

$$= (1 - \theta)t_1 - \theta t_t$$

$$= \frac{t}{1 - \theta}$$

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$$t_{t+1} = \frac{M_t (1 - \theta) t_1 - \theta t_t}{1 - \theta} = \frac{M_t (1 - \theta) t_1}{1 - \theta} - \frac{\theta t_t}{1 - \theta}$$

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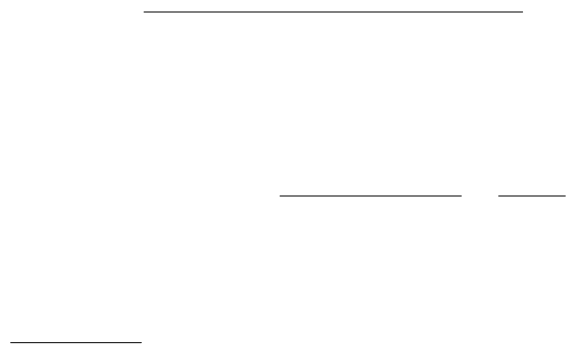
$\{t_t\}_{t=0}^T$

so the sequence  $\{t_t(1 - \theta)\}_{t=0}^T$

$$x = \max_{x \geq 0} \max_{t \geq 0} t$$

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$$E[(V - P)((S - P_t)x + (1 - t^*)x|V = S)] + (1 - t^*)[V] - P_t \quad \gamma S^2 \quad 2 \quad 2 \quad 2 \quad 2$$

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We can express PIT's optimal allocation to the stock,  $x_t$ , in terms of the average investor's optimal allocation,  $x$

$$\frac{1}{t} x_t + \frac{E_t[V] - P_t}{E_t[V] - P_t} x = \frac{1}{t} x + \frac{E_t[V] - P_t}{E_t[V] - P_t} x$$

$$x_t = t^* S - E_t[V] + (1 - t^*) \gamma S^2 + (1 - t^*) x^*$$

This expression has an intuitive interpretation: PIT's demand for the stock is  $\frac{1}{t^*}$  increased relative to that of other investors both by the potential excess return (the  $\frac{1}{t^*} (S - E_t[V])$  term), and by the implicit reduction in the riskiness of the

stock (the coefficient on  $x^*$ ). It will also be useful to have an explicit expression for PIT's excess demand, relative to that of an average investor, so define

$$\Delta_t = x_t - x^* = \frac{1 - t_t}{t} \left[ \frac{1 - t_t}{t} x_t - x^* \right]$$

$$= \frac{1 - t_t}{t} \left[ \frac{1 - t_t}{t} x_t - x^* \right] + t_t(1 - t_t) x^*$$

$$= \frac{1 - t_t}{t} \left[ \frac{1 - t_t}{t} x_t - x^* \right] + t_t(1 - t_t) x^*$$

$$= \frac{1 - t_t}{t} \left[ \frac{1 - t_t}{t} x_t - x^* \right] + t_t(1 - t_t) x^*$$

Note that since the sequence  $\{t_t\}$  is strictly decreasing, PIT's expected gain from holding his extra shares from period  $T$  to  $T+1$  strictly decreases as  $T \rightarrow \infty$ . Thus if PIT does not want to hold the extra share at date  $T$ , then ex-ante, he will not want to hold it in an period after  $T$ , so he should sell the extra share in the first period in which it is myopically optimal to do so. Furthermore, we showed that  $\lim_{t \rightarrow \infty} t_t = 0$ , so it follows our expression for  $\Delta_t$  that  $\lim_{t \rightarrow \infty} \Delta_t = 0$ . This means that PIT will not hold any extra shares indefinitely in the absence of a public revelation of  $S$ .

Assuming that the noise traders' effects on price are sufficiently small, then PIT's optimal allocation  $x_{t+1}$  will decrease every period. This means that at time zero, he will make a large purchase, and in each subsequent period, he will sell some portion of the stock that he holds in excess of  $x^*$ . If the signal turns out to be false, then PIT will gradually unwind his excess position over the rest of time. If, however, the signal turns out to be true, then PIT will sell all of his excess stock when the signal is publicly revealed. In subsequent models, we will want to examine the disposition behavior of investors, that is, where their behavior falls on the axis with the disposition effect (sell winning stocks and hold losing stocks) on one end, and the antidisposition effect on the other (hold winning stocks and sell losing stocks). It is not entirely obvious how the disposition effect should be analyzed in a theoretical setting, and one of the contributions of this paper is the introduction of methods applicable to this problem. However, even the methods developed later in this paper can be applied unmodified to the current ultra-simple model. The problem arises from the fact that PIT trades the single stock every period, so there is no meaningful notion of "unrealized paper losses" or "unrealized paper gains."

For the present analysis, I will make the ad hoc assumption that PIT can only trade in even-numbered periods. As a result, we mechanically get periods in which PIT does not sell, and so we get meaningful notions of unrealized gains and losses—the paper gains and losses recorded on the non-trading days. This

<sup>2</sup>Although this behavior of "sell when the signal arrives" is intuitively obvious, it is comforting that it is also predicted formally by the model.

mechanism is very crude, but it is readily verified that the qualitative behavior of our current simple model is invariant to the imposition of deterministic nontrade days. Now, notice that in the periods before the signal is revealed to the public, the market price follows a white-noise process centered about  $P_0$ . Since PIT sells in every period that he can—i.e. every even-numbered period—the net (assuming that he made his initial purchase at a price of  $P$ ) we would expect his paper returns before the signal revelation to be half gains and half losses, and likewise his realized returns should be half gains and half losses. As a result, in the periods before a public revelation of the signal, PIT's proportion of losses realized will be equal to his proportion of gains realized (in expectation), so there is neither a disposition effect nor an anti-disposition effect. However, when PIT's signal is publicly announced, he will sell a winning stock, and thereafter cease trading. Thus whenever the signal is true, we expect PIT to have a strictly greater proportion of gains realized than of losses realized. As long as the signal is true with strictly positive probability, PIT will exhibit a disposition effect in expectation.

In subsequent sections, I show that a disposition effect for our PIT will translate into a diluted but positive disposition effect among average investors, so that PIT's disposition behavior is potentially observable from aggregate data, even if we do not know PIT's identity.

#### 1.4 Transaction Costs and "Chunky" Trading

Adding small transaction costs to our simple baseline model not only makes the model slightly more realistic, but it also serves to tie up some technical loose ends. First, transaction costs can introduce the "non-trading days" essential to disposition behavior calculations in a very natural way. Indeed, if PIT rebalanced his portfolio only, say, every 10 periods, the disposition effect discussed in the preceding subsection would be numerically larger. Second, the imposition of small transaction costs ensures that PIT will effectively unwind his concentrated position in finite time if there is no public revelation of the signal that he observed. Small transaction costs will also yield some technical benefits in richer models. In the remainder of this subsection, I explicitly calculate the minimal level of transaction costs that would dissuade PIT from rebalancing his portfolio every period.

In the absence of the arrival of new information,  $E_{M,t+1} [V] = E_{M,t}$

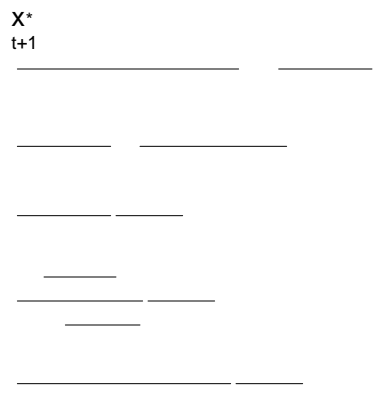
$$= P_t. \text{ Then}$$

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[V], and suppose we make the further simplifying assumption that, barring the revelation of PIT's signal,  $P$

To be rigorous, we may suppose that there is no trading at all in the odd periods, but that the market price from the previous day is recorded.

$$\begin{aligned}
& M_{t+1} \\
& S - E1 - t_{t+1} x \\
& = \frac{V_{t+1} \Delta t}{1 - t_{t+1}} \left( \frac{1 - t_{t+1}}{1 - t_t} \right) \left( \frac{1 - t_{t+1}}{1 - t_t} \right) \left( \frac{1 - t_{t+1}}{1 - t_t} \right) \dots \\
& \Delta_{t+1} \\
& = \frac{(1 - \theta) t_{t+1} - \theta t_t}{1 - t_{t+1}} \frac{1 - t_t}{1 - t_t} \Delta t \\
& 1 - \theta t_t \\
& = \frac{(1 - \theta) t_t - (1 - \theta) t_t}{(1 - \theta) \Delta t}
\end{aligned}$$



So we have the clean result that PIT wishes to reduce his excess holdings by fraction  $\theta$  in the next period, if his signal was not revealed to the public this period. We now ask, what transaction cost would make PIT indifferent between optimally rebalancing next period, and not rebalancing next period? Long calculations (see Appendix) reveal that the gain in expected utility from optimally rebalancing in the next period is

This means that a transaction cost of at least  $x_{t+1} \theta t_t (S - P) \Delta t$  would suffice to persuade the investor not to rebalance in the next period. Note that the expected gain from rebalancing increases monotonically as the number of periods between rebalancing increases. This follows immediately from the fact that the introduction of transaction costs, which a rational PIT rebalances, can be chosen small enough that the problem is interesting and tractable. In this setting, a PIT will only trade if the expected gain is not engaged in incremental trading. This is a gaining intuition with the risk.

## 2 One PIT, One Period

In this section, I expand upon the previous section, supposing that PIT sees a signal in the first observation.

of the stock's fundamental value. This small addition generates a variety of interesting results.

## 2.1 Market Structure

The market structure is essentially the same as in the first model. Again, time is discrete, indexed by  $\{0, 1, 2, \dots\}$ . There are two assets, a risk-free asset, with a return normalized to zero, available in perfectly elastic supply ("cash"), and a risky asset ("stock"), available in net supply normalized to unity. The market consists of a large number,  $N$ , of homogeneous investors, each with mean-variance utility over the fundamental value of next-period wealth, given by

$$U(W_{t+1}) = E_t [W_{t+1}] - \frac{\gamma}{2} \text{Var}_t [W_{t+1}]$$

where  $P_t$  is given by  $P_t = P_t$ . Investors myopically optimize their portfolios with respect to this utility function in each period. Finally, as before, there are some noise traders active in each period, and they demand (or supply) some random quantity  $d$  inelastically, so that residual supply is  $1 - d$ , and the market price is  $P_t/(1 - d)$ . The one difference in market structure in the present model lies in the fundamental value of the stock. As in the first model, the fundamental value of the stock is  $V$ , but now I decompose  $V$  into two components,  $V = F + \eta$ .

## 2.2 Baseline Information Structure

The fundamental value of the stock is  $V = F + \eta$ , where  $\eta \sim U[-h, h]$ , independent of  $F$ , and the values of both  $F$  and  $\eta$  are drawn before the model begins. At time zero, PIT receives a signal,  $S$ , which he alone observes. With probability  $\tau < 1$ ,  $S = F$ , and with probability  $1 - \tau$ ,  $S$  is completely uninformative. Again, I will refer to a signal of the former type as true, and one of the latter variety as false.

At the beginning of each date  $t = 1$ , with some fixed probability  $\theta > 0$ , the true fundamental value  $V$  is publicly revealed, if it has not already been revealed in an earlier period (i.e. the public revelation of  $V$  follows a Poisson arrival process). As in the first model, when the fundamental value  $V$  is publicly revealed at time  $\tilde{t}$ , market expectations change and all uncertainty is resolved,

so  $E_{M^t}[V] = V$  for  $t \geq \tilde{t}$ . The non-PIT investors believe at each time  $t < \tilde{t}$  that

$$E_{M^t}[V] = F + \tau \eta$$

and

$$\text{Var}_t[V] = \tau^2 h^2$$

follows some distribution with mean  $E[V] = F + \tau \eta$  and variance  $s_V^2 = s_F^2 + \tau^2 s_\eta^2 = s$ .

Finally, note that PIT can no longer necessarily determine whether or not  $V$  has been publicly revealed simply by observing the price. For example, if  $\eta < 0$  and  $|\eta| > F - P$ , then even upon the public revelation of  $V$ , the price of the stock will actually drop; if PIT were only looking at the stock's price, he would not necessarily be able to distinguish this price drop from a noise trader's shock. However, for the moment, we continue to assume that when  $V$  is made public, PIT directly observes this announcement, so he can still determine with perfect accuracy whether or not his signal has been made public in a given period. As

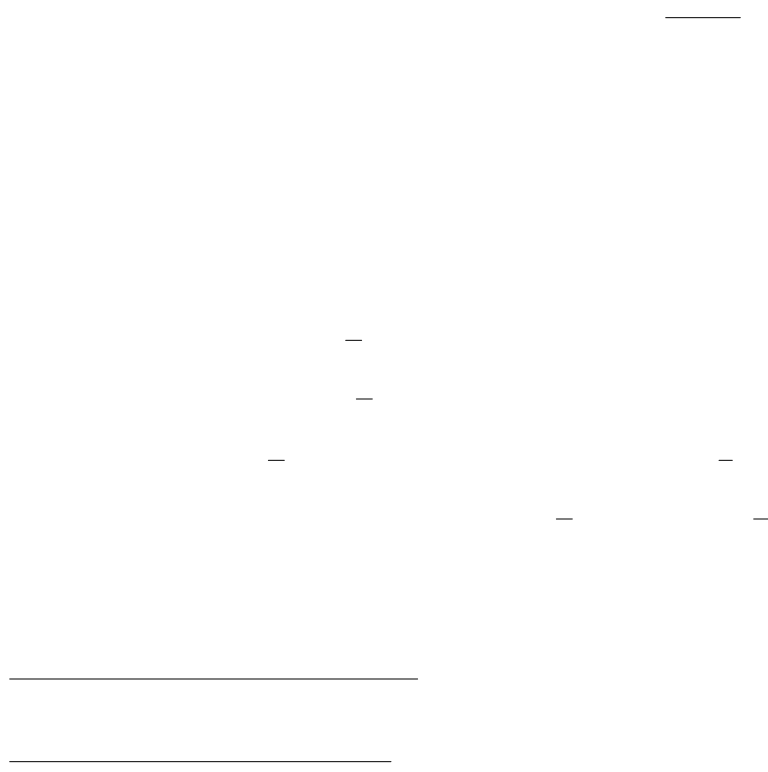
As a result, PIT still updates his estimate of  $t$  using the formula  $t_{t+1} = \frac{(1-\theta)t_t + \theta x_{t+1}}{1-\theta}$

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$$\gamma \text{Var}[V - P_t] x^2 + \frac{1}{2} s^2 x^2$$

$$[V - P_t]^2$$

$$x^2 \gamma s^2 + (1 - t_M) E[V - P_t] x - \frac{1}{2} s^2 x^2$$

$$\max_x \{ (1 - t_M) E[(F + \eta - P_t)x] - \frac{1}{2} \gamma \text{Var}[F + \eta - P_t] x^2 \} - (1 - t_M) \gamma s^2 x^2$$

$$FOC: t_M x = 0$$

$$\frac{t(S - P_t) + (1 - t) E[V - P_t]}{(1 - t_M) s} = \frac{\gamma s^2 x}{2}$$

As in the first model, this expression for PIT's optimal position in the stock has an intuitive interpretation. PIT's demand for the stock is increased relative to that of other investors both by the potential excess return (the  $t(S - P_t)$  term) and by the implicit reduction in the riskiness of the stock (the  $-t$  term in the denominator). As we would expect, PIT's optimal position in the stock is slightly smaller in the present model than it was in the first model, because the variance of the stock's fundamental value conditional upon the signal being true is now higher than it was before—

### 2.2.2 PIT Trading Behavior Under the Baseline Information Structure

This setting induces trading behavior on the part of PIT which is fundamentally very similar to that exhibited in the first model. Now, however, we get much more plausible exit behavior, because PIT will occasionally realize a loss for informational reasons. On average, when PIT exits his concentrated position because his signal has been publicly revealed, he will do so at a gain (this follows mechanically from the fact that  $\eta$  has a mean of zero), and so PIT will still exhibit a disposition effect on average. However, it is now possible for PIT to lose money even when he trades optimally on a true signal.

instead of zero.

## 2.3 Enriched Information Structure A

In this subsection, I enrich the baseline information structure introduced in the preceding subsection by splitting the revelation true fundamental value into two stages. As in the baseline structure, the fundamental value of the stock is  $V = F + \eta$ , with  $\eta \sim U[-h, h]$ , independent of  $F$ , and the values of both  $F$  and  $\eta$  are drawn before the model begins. Similarly, at time zero, PIT again receives a signal,  $S$ , which he alone observes. With probability  $t_0 < 1$ ,  $S = F$ , and with probability  $1 - t_0$ ,  $S$  is completely uninformative. In each period, beginning at time  $t = 1$ , according to a Poisson arrival

process with parameter  $\theta$ , all investors learn observe  $\eta$ . When this occurs, say at time  $\tilde{t}$ , market expectations update so that

$E_t[V] = E_t[F] + \eta = E[V] + \eta$  where the equality of  $E_t[V]$  and  $E_t[F]$  follows from the fact that  $\eta$  has an ex-ante expected value of zero. When  $\eta$  is revealed, the average investor's uncertainty about  $V$  decreases from  $s_{2F} + s_{2\eta}$  to  $s_{2F}$ , and PIT's uncertainty about  $V$ , conditional upon his signal being true, decreases from  $s_{2\eta}$  to zero. As a result, the new market price (ignoring noise) becomes

$$P_t = N_t E_t[F] + \gamma s_{2F} [V] + \eta$$

Following the revelation of  $\eta$  at time  $\tilde{t}$ , the public revelation of true value of  $F$  begins to follow a Poisson arrival process, again with parameter  $\theta$ , so we return to a version of the original, single-signal model. Before  $\tilde{t}$ , PIT does not update his estimate of  $t$ , but after time  $\tilde{t}$ , he begins to update it in the familiar manner.<sup>4</sup>

### 2.3.1 Empirical Implications Like the baseline information structure, Enriched Information Structure A (EIS-

A) induces more plausible exit behavior by PIT, but EIS-A also delivers a stark empirical prediction. In the days and weeks following an unanticipated arrival of bad news for a particular stock (which corresponds to the realization of a large negative  $\eta$ ), we should expect to see a reduction in the disposition effect among investors who purchased the stock before the bad news. In the language of the model, if  $\eta$  is so negative that  $F + \eta < P_5$ , then we should expect to see PIT exit his concentrated position at a price below his initial purchase price. However, by analogous logic, we should not expect to see any reduction in the disposition effect among investor who purchased the stock after the bad news arrived.<sup>5</sup>

<sup>4</sup>There is no theoretical reason why we could not begin the  $F$ -revelation process at time zero, whereupon PIT would be updating his estimate of  $t$  immediately, but the analysis is more complicated, so this alternative is deferred to a later section.

<sup>5</sup>I am grateful to Andrei Shleifer for pointing out this empirical implication.

## 2.4 Enriched Information Structure B

The fundamental value of the stock is now simply  $V = F$ , with  $F \sim N(\mu, s_2 v)$ . As before, the value of  $F$  is drawn before the model begins. At time zero, PIT receives a private signal  $S$  of the usual form, where  $S = F$  if the signal is true, and  $S$  is uninformative otherwise. At time  $t = 1$ , a noisy signal of  $V$ , call it  $\Psi$ , is publicly revealed. Assume that  $\Psi = V + \epsilon$ , where  $\epsilon \sim N(0, s)$  and  $\epsilon$  is independent of  $F$ . Then, in each period  $t$ , the fundamental value of the stock is potentially publicly revealed according to the usual Poisson arrival process.

2.4.1 Investors' Belief Updating Let  $\psi$  denote the realized value of  $\Psi$ ; following this revelation, non-PIT investors update their priors and assign  $V$  a posterior (Gaussian) distribution with the following moments:

$$E[V | \Psi = \psi] = \mu + \frac{s_2 v}{s_2 v + s} (\psi - \mu)$$

$$= \mu + \frac{s_2 v}{s_2 v + s} (\psi - \mu)$$

$$E[V | \Psi = \psi] = \mu + \frac{s_2 v}{s_2 v + s} (\psi - \mu)$$

$$V \text{ arm } [V | \Psi = \psi] = \frac{1}{\frac{1}{s_2 v} + \frac{1}{s}}$$

$$= \frac{s_2 v + s}{s_2 v + s} \left( \frac{s_2 v}{s_2 v} + \frac{s}{s} \right)$$

$$= \frac{s_2 v + s}{s_2 v + s} \left( \frac{s_2 v}{s_2 v} + \frac{s}{s} \right)$$

$s_2 + s$

The updating problem facing PIT is clearly different, since his prior beliefs about  $V$  differ from those of the other investors. PIT uses the signal  $\psi$  to update his estimate of the probability that his signal is true. If PIT's signal is true, then  $\Psi \sim N(F, s)$ , while if his signal is false,  $\Psi \sim N(\mu, s_2 v + s_2)$ , so we have the following conditional densities for  $\Psi = \psi$ :

$$g(\psi | \text{true}) = \frac{1}{\sqrt{2\pi s}} \exp\left(-\frac{(\psi - F)^2}{2s}\right)$$

$$g(\psi | \text{false}) = \frac{1}{\sqrt{2\pi(s_2 v + s_2)}} \exp\left(-\frac{(\psi - \mu)^2}{2(s_2 v + s_2)}\right)$$

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$$\exp - \frac{(\psi - F)^2}{2s^2}$$

+ s<sup>2</sup> V

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$$\exp - \frac{(\psi - \mu)^2}{2(s^2 + \frac{s^2}{V})} (\psi - F)$$

1 + s<sup>2</sup>  
V s<sup>2</sup>

$$\frac{1}{\sqrt{2\pi(s^2 + \frac{s^2}{V})}} \exp - \frac{(\psi - \mu)^2}{2(s^2 + \frac{s^2}{V})} \frac{2(s^2 + \frac{s^2}{V})}{2s^2} (\psi - F)$$

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$\frac{2}{V}$

$$\frac{2s^2}{2(\psi - F)^2} (\psi - \mu) + (\psi - \mu) + s$$

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$$t_1 = \frac{\psi t_0}{\psi t_0 + (1 - t_0)}$$

$\psi > 1$ .

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$$= E_{M1} \gamma V \frac{[V]}{P} \frac{1}{[V]}$$

$$= \mu + \frac{s^2}{V} (\psi - \mu) - \frac{s^2 V}{P s} \frac{1}{(\psi - \mu) \gamma s^2 s^2 V}$$

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 $s$   
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 $2$

$$= s^2 \mu + s (\mu - P_1) + s P_1$$

$$\frac{s^2 \psi - s^2}{V} \frac{s^2 V + s}{V}$$

$$\frac{s^2 \mu + s^2 V \psi}{\gamma s^2 s^2 V} \frac{\gamma P_1}{s + p \gamma P}$$

$$= s^2 \frac{\mu + s^2 V \psi}{\gamma s^2 s^2 V} \frac{s^2 + s^2 V}{s^2} \gamma =$$

$s$      $1$   
 $2$   
 $V$











$$\begin{aligned}
 & -t \quad \psi \\
 & \wedge t + (1-t) \quad 0\psi \\
 & \vee P \quad \wedge \quad + (1-t) \quad \psi t_0 \quad 0 \\
 & \quad \quad \quad t_0 \quad 1 \quad p_V \\
 & \quad \quad \quad ) \quad 1 - \quad 0_0 \quad 0 \\
 & p_V \quad P = (\wedge t_0 \psi^1 \quad + (1 - t_0) t_0 \quad ) (1 - t_0 + p) = p_V \vee P P_0 1 \wedge \psi_0 \\
 & \quad \quad \quad 16 \\
 & \quad \quad \quad + (1 - t \quad \psi t_0 \quad \quad (1 - t_0 + p \quad 0
 \end{aligned}$$

$x_{t+1/P}$

$$\frac{s_2^2 (\psi - \mu)^2}{2 (s_2^2 + s_1^2 V)} (\psi - F)^2 \frac{p + pV}{pV} \frac{P_0}{P_1}$$

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$x_{t+1/P}$   
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s

$+ s_2^2 V_s$ ) «I am grateful to Andre i Shi

This expression gives very clean insights into PIT's trading behavior. When price increases, the potential reward if PIT's signal is true is relatively smaller, so ceteris paribus this reduces the number of shares that PIT optimally wishes to hold. This is the standard disposition effect in its purest form. However, the public signal will also cause PIT to update his estimate of the probability,  $t$ , that his private signal was true. If the public signal provides evidence that PIT's private signal was true (which manifests as the likelihood ratio term,  $\Lambda_{x_{t+1/P}}$ , in the expression for  $x_{t+1/P}$ ), then this tends to increase the number of shares that PIT wishes to hold. Finally, as the precision of the public signal increases relative to that of PIT's private signal, both of the aforementioned effects are magnified.

2.4.3 Empirical Predictions Enriched Information Structure B (EIS-B) predicts that disposition behavior will be affected in a predictable manner by the relative reliability/precision of public vs. private information. If we can identify stocks about which we expect an investor to have more reliable private information, then we should see an increased disposition effect by that investor for those stocks, relative to his other holdings. A distinct empirical implication of this arises among mutual fund managers, since we can use some measure of "proximity" as an instrument for more reliable private information. It is well documented that mutual fund managers tend to make abnormal returns on stocks of firms that are close to them physically, or close to them in a "social network of managers" sense, (cf. Hong, Kubik and Stein (2005) and Cohen, Frazzini and Malloy (2008)) so the extant literature contains measures of proximity suitable for our purpose.

### 3 One PIT, One Private Signal, Multiple Public Signals

#### 3.1 Model Specification

Assume that the market structure is the same as it has been in all preceding models. The information structure is very similar to EIS-B, but now there is a sequence of noisy public signals of the stock's fundamental value. More specifically, the fundamental value of the stock is given by  $V = F$ , with  $F \sim N(\mu, s^2)$ . As before, the value of  $F$  is drawn before the model begins. At time zero, PIT receives a private signal  $S$  of the usual form, where  $S = F$  if the signal is true, and  $S$  is uninformative otherwise. At time  $t = 1$ , a noisy signal of  $V$ , call it  $\Phi_1 = V + \epsilon_1$ , is publicly revealed. Assume that  $\Phi_1$ , where  $\epsilon_1 \sim N(0, s_1^2)$  and  $\epsilon_1$  is independent of  $F$ . After  $\Phi_1$

<sup>6</sup>This expression can be further expanded using the definition of the likelihood ratio:



is independent of both 1

$\sim$  i.i.d.  $N(0, \sigma^2)$ , and each  $\mu_i$  and  $F$ . Since the error terms in each signal are independent and have uniformly bounded variance,  $E[V | F_t] \rightarrow V$  a.s. Thus there is no need to assume that the stock's true fundamental value is ever perfectly revealed as a Poisson arrival in this information structure, although we could introduce this feature if desired.

$$= \frac{N}{S} \frac{\sigma^2}{\sigma^2 + S^2 V^2} \mu + \frac{S^2 V^2}{\sigma^2 + S^2 V^2} V$$

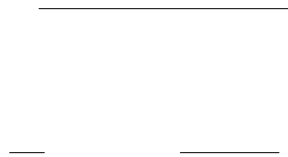
### 3.2 Belief Updating and Trading Behavior

All of the calculations and results from EIS-B extend immediately in the present setting. Now the updating procedures are applied recursively, but otherwise all the previous results from EIS-B apply exactly to this multiple-public-signal structure.

### 3.3 The Punchline

$P_1$

Hence  $P_1$



Rather than assuming that every market participant literally observes a sequence of noisy signals of the stock's fundamental value, suppose that some subset of investors (i.e. much more than a single investor, so that there is no inconsistency with the assumption that  $P_1$ 's trades have a negligible impact on prices) sees a noisy signal directly and that prices are largely revealing so that the market observes the signal as an innovation in the market

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(that is, market price ex-noise) is a sufficient statistic of  $\psi$ , provided that  $s_2$  is known;  $\psi = \frac{ss_2^2}{V(P_1 - \mu) + N + \gamma s_2^2 N P_1}$

Provided that the statistical assumptions on the  $\{u_t\}$  (i.i.d., normality, zero mean) are satisfied, then the analysis from EIS-B, as applied to the multiple public-signal setting, applies exactly. However, due to noise trader activity, PIT observes only a noisy signal of  $P$ , so he faces the additional signal-extraction problem of inferring the value of  $\psi$  from the market price. This means that the informational content of price innovations in the market for the stock now plays a role in how PIT updates his beliefs about  $t$ .

If we make this conceptual leap from public signals to price innovations, the empirical content of the model increases exponentially, because PIT disposition behavior is now a function of the informational content of price innovations (or equivalently, the degree of adverse selection) in the market for the stock. In particular, the model easily accommodates the systematic differences in the disposition effect as a function of the underlying asset market's characteristics



observed by Rangelova (2000), as well as generating many analogous empirical predictions for the covariation between disposition behavior and market characteristics that provide some measure of the informativeness of price movements.

### 3.4 Many PITs

Up to this point, I have assumed that a single investor, PIT, receives a private signal at time zero. While this assumption simplified the analysis, it is unrealistic and unnecessarily restrictive. Without loss of generality, we could suppose that PIT actually represents a collection of many small investors; the key condition is that PIT remain small enough relative to the overall market and the level of noise trader activity that PIT's trades have a negligible impact on price. Also, instead of stipulating that all PITs observe exactly the same signal, it follows naturally from EIS-A that we could introduce some idiosyncratic, mean-zero noise into the signal that each PIT observes. Since this device would reduce the tendency for all of the PITs to exit their concentrated position at the same time, it would be a useful addition to a many-PIT model.

The case in which the population of PITs grows large enough that the PITs have a non-negligible impact on prices is beyond the scope of this paper. In a working paper (Clark-Joseph (2009)), I investigate a setting closely related to this, and obtain results qualitatively similar to those in this paper, but many open questions remain, and I regard this as a promising area for future research.

## Part III

# Theoretical Treatment of the Disposition Effect

## 84 Average Disposition Effect vs. Disposition Effect Among PITs

From the perspective of testing the predictions of the model in this paper, two related questions about measurement of the disposition effect are of paramount importance:

1. If we observed the trades of all participants in the market for a given stock, could a disposition (or anti-disposition) effect be observed in aggregate, or is "disposition" conserved on a market-wide basis, like net trading profit?

<sup>8</sup>The issues discussed in this section are somewhat deep, and they are relevant to a nontrivial body of behavioral finance literature beyond this paper. It may be worthwhile to develop these ideas at greater length in a companion paper. Also, although I have limited my exposition to the simplest cases, generalizations of the issues in this section suggest a number of purely econometric questions which may be of independent interest.

2. Does the presence of (anti-) disposition effects among the PIT population for a given stock induce a corresponding effect on the average (anti-) disposition effect measured for that stock?

In essence, the model delivers clean predictions about the disposition behavior of PITs, but because we cannot observe who the PITs are, empirical assessment is feasible only if these predictions about the aggregate behavior of all investors or the behavior of the average investor. In the remainder of this section, I first develop a suitable technique for analyzing the disposition effect in a model such as my own, then I use this technique to address the two questions posed above. In short, I demonstrate that under mild regularity conditions, 1) disposition behavior can be aggregated over an entire market to provide a meaningful statistic, and 2) the disposition behavior of the PIT population will generally induce corresponding aggregate disposition behavior.

#### 4.1 Characterizing the Disposition Effect in a Theoretical Setting

The standard method for measuring the average disposition effect that a given investor exhibits entails taking a “snapshot” of that investor’s portfolio each time that he sells stock, and marking this as a “realization” event. At the date of a realization, the stocks in the investor’s portfolio are characterized as either “winners” or “losers,” depending on whether they are above or below their purchase price, respectively, as are the stocks that the investor bought or sold; these data are then used to calculate the “proportion of gains realized” (PGR) and the “proportion of losses realized” (PLR). Similarly, by aggregating these four measures both over time and across investors for a single stock, following Rangelova (2000), this method can be adapted to measure the average disposition effect among investors in that stock.

In empirical applications, an important obstacle is presented by the fact that investors may face unobserved factors which influence the times at which they choose to sell stock, and those unobserved factors could be correlated with price movements. Recording measurements at the dates up on which realizations occur helps to deal with this problem, so the technique is important for empirical work. However, this “realization-dated sampling” obscures the theoretical relationships between measurements of disposition effects and underlying investor trading behavior. For theoretical analyses, the cumbersome machinery of realization-dated sampling is unnecessary, and in its place we can simply sample every period. The PLR and PGR that result from every-period sampling (call these PLRE and PGRE, respectively) will clearly be smaller in absolute magnitude than their respective analogs calculated via realization-dated sampling, but the ordering will be preserved; i.e. if  $E[PLR] > E[PGR]$  then  $PLRE > PGRE$ , and if  $E[PLR] > E[PGR]$ , then  $PLRE < PGRE$ . The quantities PLRE and PGRE are always defined with respect to the same sequence of sampling dates (i.e., every period), so their expectations can be computed directly from the trading rule and price-process for a given stock.

By contrast, computing the expectations of the traditional PLR and PGR requires knowledge of the trading rules and price-processes of every stock in the investor's portfolio. As a result, if different investors hold different portfolios, PLRE and PGRE naturally lend themselves to cross-stock comparisons, whereas expectations of PLR and PGR do not.

In short, PLRE and PGRE serve as the analytical tools necessary to address the questions raised at the beginning of this section.

## 4.2 Can an Aggregate Disposition Effect Exist?

To illustrate that (and how) an aggregate disposition effect can exist, consider a stock with a single, indivisible share, and assume that the stock's log-price,  $p$ , follows a simple, symmetric random walk on the integers. Consider a single investor,  $d_0$ , who purchases that stock at date  $t=0$  at a log-price normalized to zero. Let  $t$  denote the first date at which  $p=2$ , and let  $\hat{t}$  denote the first time at which  $p=-3$  ("gain" and "loss," respectively). The investor where  $\hat{t} = \min\{t_{g0}, t_{l0}\}$  trades according to the following strategy: sell at time  $\hat{t} \in \{1, 4\}$ . Since  $\hat{t}$  is a stopping time and the log-price process is a martingale, the log-price process stopped at  $\hat{t}$  is a martingale, so the investor's expected sale price is equal to his purchase price—on average, the investor breaks even by following this trading rule. Intuitively, in the first period after purchase, the investor sells the stock either when its log-price hits an upper target, or when its log-price falls so low that the investor decides that his initial assessment was mistaken; in period 4, the investor no longer thinks that he has any meaningful information about future price movements, so he sells the stock at the current market price, if he has not done so already. Note that this trading rule is qualitatively similar to a strategy that a PIT might follow. Using the specified trading strategy, we can calculate the number of total gains (sampling every period), total losses (sampling every period), realized gains, and realized losses along each possible sample path of the stopped log-price process.

When does  $d_1$  sell the stock, a different investor, call him  $d_1$

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$t_1$   
follows the same trading strategy as did  $d_0$ .  
Then the setting for  $d_0$

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$d_1$ , must purchase it. Gains and losses for  $d_1$  are determined relative to the log-price at which he buys the stock from  $d_0$ , so without loss of generality, we can normalize this initial log-price to zero. Clearly, without loss of generality, we can also re-normalize the date so that the period in which  $d_1$  initially buys the stock is labeled as zero. Now, suppose that  $d_1$ , that is, he sells at the (renormalized) time is identical to the setting for  $d_0$ . If we continue in this manner for a sequence of investors who all follow the same trading strategy,  $\{d_j\}$ , with  $d_j$  selling the stock to  $d_{j+1}$ , we will get the

By introducing this sequence of investors, we can also justify our assumptions about the price process. Suppose that at each date, there are two identical investors in the market in addition to the one who owns the stock, and these other investor offers to buy the stock at some price (they compete via Bertrand competition, so their offers are driven to exactly their willingness to pay; the stock is allocated to one of them at random). If we define this offer to be the market price in that period, then we can generate any price process that we desire through an appropriate choice of stochastic process for the beliefs that these other investors



sequence  $\{(\#total\ losses, \#total\ gains, \#realized\ losses, \#realized\ gains)\}_{j=0}^8$

0

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$$E[\#total\ losses] = 23$$

$$E[\#total\ gains] = 18$$

$$E[\#realized\ losses] = 5$$

$$E[\#realized\ gains] = 6$$

and thus  $E[\#realized\ losses] / E[\#total\ losses] \sim 0.217$   
 $E[\#realized\ gains] / E[\#total\ gains] \sim 0.333$

that captures the trading behavior of every investor who participated in the market, and the sample average of each component (e.g. # total losses) converges to the expectation of that component for investor d. We know the stochastic process for the log-price process, so we can assign probabilities to each possible sample path that could have occurred and directly calculate the expectation for each of these components. With the given example parameter values, and using the convention that neither a loss nor a gain is recorded if sale price equals purchase price, these expectations are

$$\lim_{J \rightarrow \infty} \frac{\sum_{j=0}^J \#realized\ losses_j}{\sum_{j=0}^J \#total\ losses_j} = 0$$

, noting that

$$\sum_{j=0}^J \#realized\ losses_j = \sum_{j=0}^J \#total\ losses_j - \sum_{j=0}^J \#realized\ gains_j$$

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$$\frac{\#realized\ losses}{\#total\ losses} = \frac{\#realized\ losses + \#realized\ gains}{\#total\ losses + \#realized\ gains} - \frac{\#realized\ gains}{\#total\ losses + \#realized\ gains}$$

Of course, by Jensen's inequality,  $E[\frac{\#realized\ losses}{\#total\ losses}] \geq \frac{E[\#realized\ losses]}{E[\#total\ losses]}$  might in stead wish to directly calculate (e.g.)  $E[\frac{\#realized\ losses}{\#total\ losses}]$

=  $E[\frac{\#realized\ losses}{\#total\ losses}]$ , so we or d

The only difficulty with this approach is that the sum on the left-hand side may include terms of the form "zero/zero," so some convention must be adopted to deal with these. In the case at hand, if

$$E[\frac{\#realized\ losses}{\#total\ losses}] = 1, \text{ then}$$

$$E[\frac{\#realized\ losses}{\#total\ losses}] = 0.484$$

$$E[\frac{\#realized\ losses}{\#total\ losses}] = 0.552$$

w h i l e

$$\frac{\#realized\ losses}{\#total\ losses} = \frac{\#realized\ losses + \#realized\ gains}{\#total\ losses + \#realized\ gains} - \frac{\#realized\ gains}{\#total\ losses + \#realized\ gains}$$

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t . 175 = 0.283  
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These results suggest that the details of the averaging scheme may be important for empirical work, but they also demonstrate that an aggregate disposition effect can exist. Moreover, it follows from the symmetry of this example that an aggregate anti-disposition effect could also exist. Because the reference point for gains and losses resets when the stock changes hands, disposition behavior need not “wash out” on average, so average/aggregate disposition behavior is a meaningful quantity. This is also true because PLRE and PGRE are pricepath-dependent, so these measures of disposition behavior are not conserved in the mathematical sense. Finally, although the preceding analysis employed an infinite sequence of investors, note that it suffices to have just two investors whose beliefs about the value of the stock may differ and vary over time.

### 4.3 Will a Disposition Effect Show Up for the Average Investor, or Only for PITs?

The analysis in the preceding subsection can be extended to characterize the behavior of the measured “average” disposition effect when investors do not all follow the same trading strategy.

Consider the example from subsection 4.2, but now suppose that on each odd-numbered transaction, a  $d$ -type investor, say  $d_{2j+1}$ , sells the stock to a new type of investor, a “ $q$ -type” investor, say  $q$ . All of the  $d$ -type investors follow the trading strategy discussed earlier; they sell the stock at the stopping time  $\hat{t}$ . The  $q$ -type investors, however, follow a different strategy; they hold the stock for  $h$  periods, and then sell it, regardless of the market price. The  $q$ -type investors can be thought of as passive investors. Although this setting seems different from that of the first example, the market still has a repeated structure. Thus if we consider blocks of market observations, each of which covers one  $d$ -investor and one  $q$ -investor, analysis proceeds exactly as it did in the first example.

By a symmetry argument, it follows that in expectation,  $PLRE = PGRE$  among  $q$ -type investors. Consequently, the difference  $PLRE - PGRE$  will be closer to zero when measured over the entire population of investors than it would be if measured only over  $d$ -type investors, but the expected sign of this difference will not change as a result of introducing the  $q$ -investors. By varying the ratio of  $q$ -investors to  $d$ -investors (and noting that their order in the trading sequence doesn’t matter, as long as their relative distribution preserves the ergodicity of the desired sample averages), and by varying the holding period  $h$  of the  $q$ -investors, the relative influence of the passive investors can be made arbitrarily large or small, and the same conclusions hold. Thus the empirical predictions of the model in this paper should be observable in average measures of disposition behavior, even though the model only predicts effects among the PIT population.

<sup>10</sup>It may be interesting to examine the properties of aggregate disposition behavior when there is richer variation among investor types, say some disposition investors, and some antidisposition investors, but that matter is beyond the scope of this paper.

## Part IV

# Conclusion

In this paper, I have developed a novel model to explain and analyze disposition behavior, as well as some new theoretical results on the link between empirically observed average disposition behavior and the disposition behavior that my model (or any other) predicts for a single investor. My model demonstrates that a simple but reasonable specification of gradual information diffusion can explain many observed stylized facts about individuals' trading behavior in asset markets. In particular, the model generates trading behavior that spans the axis from "disposition effect" behavior on one extreme, to "momentum chasing" on the other extreme, without recourse to behavioral explanations, and so it rationalizes the empirical findings that trading varies along this disposition-momentum axis according to the characteristics of the underlying asset market. The model also suggests several other testable implications derived from its underlying "price-target" framework, and empirical investigations of these implications constitute promising avenues for future research.

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