Ensemble Learning for Reectometry

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TR-06-10

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Technical report accompanying \Blind Reectometry", in Proc. European Co nference on Computer Vision, 2010.

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Abstract. In \Blind Reectometry" (Romeiro and Zickler, 2010

[3]) we describe a variational Bayesian approach to inferring material

properties (BRDF) from a single image of a known shape under unknown, r eal world illumination. This technical report provides additional detail s of that approach. First, we detail the prior probability distribution for natural lighting environments. Second, we provide a derivation of the bilinear likelihood expression that is based on discretizing the re ndering equation. Third and nally, we provide the update equations for the iterative algorithm that computes an approximation to the posterio r distribution of BRDFs.

1 Illumination Representation Details

As per Eq. 5 in [3], the prior pr

obability distribution on the wavelet coe cients of real-world illumination environments is of the form

$$p(\ell) = \prod_{m=2}^{M} \sum_{n=1}^{N_m} \pi_{nm} \, p_{nm}(\ell_m); \quad (1)$$

with N_m the number of mixture components for coe cient m, and π the mixing weights. All coe cients in any one group (de ned in Sect. 3.1 [3]) share the same N_m, π_{nm} and p_{nm}. The exact forms of the mixture components are as follows:

p_{nm} (N(0;v);if m6= 3 N_{RC} $_{nm}$(‘$_{nm}$;v_{nm};T);if m= 3 ; (2)

v_{nm} = 8 >v< >:$_{nr(m)}$ $_{g(m)}$; if r(m) 2f2;3g $v_{nr(m)}$; if r(m) 2f4;5g v_{nm}; otherwise and N_m = 8 >r(m); if r(m) 2f3;4;5g 2; if r(m) 2f1;2g;m6= 3 1; if m= 3< >:

where m = 3 is the w

avelet coe cient corresponding to the vertical basis element at the coarsest scale, $r(m)$ indicates the scale of coe cient m, and $g(m)$ in dicates to which grouping (spatial location and basis element type) coecient m belongs to (see Fig. 2 in [3]). N_{RC} corresponds to a Gaussian distribution recti ed at a given point. x N_{RC} Letting the recti cation point be T, say, the recti ed Gaussian distribution has the form(u;w;T) $p(x) = 8< p 2p werfc((uT) p2w)exp(xu)2w2; if x T 0; otherwise and it is related to the \standard" recti ed gaussian distribution by x N(u;w) $x N_{RC};(3)(u;w;0): (4)

Furthermore, if x N_{RC}(u;w;T), then

= u+ p2w p erfcx((uT) p2w

hxi p(x)

2X p(x)

p ( ) i i r i(!)Fi(!)(ni m !)d!+ ; (5)

P ;F f (Wif)

i) = uhxi+ w+ T(hxiu); where h irepresents the expectation under distribution p( ).

2 Discrete rendering equation

We de ne the following imaging model (Eq. 7 in [3]):

I = Z L (!)V r(!) and Fi(!) P ‘ r m r

i

i

k k

(Ri‘)

i i (!)Fi(!)(ni r1;r2;r3 (Ri‘)r1(W if)r2(vi) (!)V L 3C 1;r2;r3; (6)

Z

r D

Cr 1;r2;r3 = Z !)d! r2(!) r3(!)(n i (!) r1

i i i)r r i !)d!if r1 = r r

= r3

r 2

(v

D r 1

0, otherwise

with ;;Li;V D= (R (n being the noise, exposure, local lighting, local visibility, and reectance, respectively.Given linear lighting and reectance representations, L = Pand F=, we can write Li(!) P(!), where f (!)gis the delta basis in a discretized local hemisphere (a discretization of in Eq. 5), Ris the linear transformation that maps from the wavelet basis in domain of the octahedral map into the delta basis in D(see [4]), and Wis the linear transformation that maps from the NMF basis into the delta basis in D. We also represent the visibility in the delta basis in the discrete hemisphere (V(!) P(!)), and substitute these into the rendering equation: !)d!

X

(Ri‘)r(Wif)r(vi)rCr; (7)

where Li(!)Vi(!)Fi(!)(ni (!). Thus, is the support of r1 Zr !)d! X

D r 1

which can be written in matrix form as ‘T Mif, where the per-pixel matrices Mi = RT i

q Ii diag(C vi)Wi Mi R k;w); (9)

= ‘T

re given by Mi, where is the Hadamard (or entrywise) product, and diag( ) is a square matrix with its argument along the diagonal. Substituting this expression into Eq. 5, we have

f+ ; (8) which is exactly Eq. 8 in [3].

3 Update equations for the posterior approximation

As mentioned in Sect. 3.4 of [3], the ensemble of distributions q( ) that approximate the posterior distribution of reectance f, lighting ‘, and exposure and noise parameters ; are of the forms

m(‘m); with q m (N(um) if m6= 3 NRC;w(u;w

); (11) q(2) (2;ap;bp); (12) where Ncorresponds to a Gaussian distribution recti ed at 0 and Ncorresponds to a Gaussian distribution recti ed at T (see Eq. 3 above). The closedform expressions for the updated parameters of each approximating distribution in terms of the current parameters of the others (Algorithm 1 in [3]) are as follows. In these expressions, the notation Mimkrefers to the mkthelement of matrix Mi.

q (u ; RC w

R C k; (13)

N i=1

q(f) = Yq(‘) = Yq() N k(fk); with qk N (u k m;T) otherwise, (10)

R m m

= 22 q(2) X ki

uk wk

Ni= 1X * (mX ‘mMi mk)2+ 2 q( ) ; (14)

1 wk

T i T q(‘)

Mif ; (15)

2

= 2= q(2

2 2)q(

i

u = 2 q(2) 2) Ni=1X2I ‘ q(‘;f)

w Ni=1X q(‘;f)

(‘

1 w

M f) ; (16)

= 22 q(2) Ni=1XEmi + NmXn=1nm ‘nm ; (17)

1 vnm

um

wm

1 wm = < 2 >q(2) Ni=1Xk<(X Mimk fk)2 >q(f)<2 >q( ) + NmXn=1nm ;(18)

1 vnm

0 @= 2 >q(‘;f) ‘ Mimk !+ ; (21)

*mXr6=m k;)

i1 A X

k

i q(‘

ap = a+ P= b+ k m< sM 2 ; (19)

NN i=1 (2 ; (20)

T

bp

with,

= 2 *

k s ‘ Mim I m q(‘;f

i 6 m

C = sf

k

X

Emi 0 X kX ‘rMirk fk Ii1 A X M mk f !+ ; (22)

@ m;f;)

nm = m exp(hlog(nmpnm(‘m))iqm(‘m)); (23)

thkcorresponds to the vector fwith its kentry removed. Simpli ed expressions for some of these expectations and formulas are in Sect. 5 below. But rst, in order to provide some examples, the next section includes derivations of the parametric forms of two of the ensemble distributions (qk(f) and q()) as well as the update equations for their respective parameters (ukk;wkand u;w) listed above.

3.1 Parametric form and up date eq uations for qk(fk)

Following Miskin [1] and Miskin and MacKay [2], we nd the form of the ensem-kk(fdqble distribution qk( ), by taking the variational derivative of Cwith respect to qk, dCKL) = logqk(fk) logpk(fk)(24)

+ 22 q(2) Ni=1X (‘T MifIi)2 q(‘;fk;) (25)

qk(fk) /pk(fk)exp 22 2 i=1X T fI) q(‘;fk;)! :

where m are such that PNmn=1 nm = 1, and the notation f KL

+ 1 + ; (26) which, when set to zero, leads to

q( ) N (‘ Mi i 2

En se m bl e Le ar ni ng for Re ec to m etr y 5

(‘TMiflI)2 We can now expand the summand, q(Ii m+ X ‘mMimk fk)2+ k;)

‘;fk)= *m(Xs6=k X‘mMims fs

‘ ! + q(‘;f

fa2 k m M i m k

= 2+ 2 0 X X ‘mMims f I 1 2 sm* X m X ‘mMimk !+ fk

* q( )m @ A q(‘;fk;)

i q(‘)

s6=k

+ ::: 2 k + C q( ) to obtain f

q = B k(f

2

+ k 0 and qk(fk(fk) NR(uk;wk), where uk;wk (29)

2

2 dCKLKLdq() = logq() logp()(28) with respect to q() we obtain

Ni=1 X (‘TMiflI)2 q(‘;f)

q (

f o r

2)

f k

ki f k i (B k 2 k + C f) ! : (27)

)

k

N

i=1

k(fk) /pk(fk)exp

+ ::: an d su bs tit ut e

this into the expression for q

2 ² X ki f ki k

Substituting $p_k(f_k)$ into this expression, we nally arrive at

q)exp 2 2 q(2) i=1 X! f 2 N k(f k) /exp(k f k (B ki f 2 k + C ki f k) ! C k! f !

k i=1 X

i=1 ki N 2) 2 2 q(= exp + 2 2 q(2) N k

ki

X B

)

= 0 otherwise.

After completing the squares, this last equation leads to qare as de ned in Eqs. 1323.

3.2 Parametric form

and update equations for q( ) Taking the variational derivativ e

o
f

C
+
1
+
;
(3
0)
(3
1)

which, when set to zero, leads to Ni= 1X ('T MiflI)2 q(';f)! :

q() /p()exp 22 q(2)

f)2 q(';f) 2 2Ii T Mi q(';f)
i=1X
i

i2 22 2

q() /p()exp

+ ::: = G + H + ::: and substitute into the expression for q( )

to
ob
tai
n

) N + Hi) ! : (32)

q((Gi2

As before, we expand = *0 @= ('s6=kXmMimsfs Ii m+ 'mMimk fk1 A2+q(';fk;)

XTmMi ' Xf

('TMiflI)2 q(';f)

R(
u;
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to arrive at q( ) N are as de n ed in Eqs. 13-23.

4 Cost function C_{KL} in Algorithm 1

To derive the expression for the cost function C_{KL} in Algorithm 1 of [3] we follow Miskin [1] and Miskin and MacKay [2] to approximate some of the terms in Eq. 11 of [3] with their upper bound. This allows us to re-write this expression as

2) KLq( ); (33) with each term as given below.

m(‘)i (34)m(‘m

M X m =2M X m=2M X m=2

m 2 (‘m) m)

m

q

1 2 log2 wm 2w 1 (‘m u) q (‘

CKL = C(‘) KL + C(f) KL + C(+ C( ) KL <logp(Dj) >

C(‘) KL = hlogqm(‘m)iq m(‘m) hlogp m q)

nm log nm m)i m m + hlogpnm(‘

Nm X n=1 n m

C(f) KL = hlogq(fk)iqk(f k) hlogpk(f)i k(fk) (35)

= 1 2 log2 wk 12wk (fk uk) q (fk)

log k k hfki qk(fk)

k q k 2

K X k=1 K X k=1 K X k=1

C
( )

2
)

K
L

= logq(2= log p logp(p+ a) q(2)2 q(2) + (aa) 2 (36
(b)(bp) q(2) + log (bp b)
b b) log p q(2)
a
hlogp(Dj)iq( ) N= N2 log2 + q(2) (38)
2
12 2 q( ) hlogp()iq( ) 12w (u) log + hiq( )
TMi q(';f)
= hlogq()iq( )1
2log2 w Ni=1X fIi2)2
C( ) KL (' log 2(37) =

5 Simpli ed expressions for Eqs. 13-23

= ('2 h'i)T M(2) i(:;k)+ (h'iT Mi(:;k))2;
* (mX'mMimk)2+q(') i
i(:;k)
t
h
kXMimkfk!2+q(f) = M(2) i (:;k)(f 2 hfi) + (Mi(m;:) hfi) :
i(k;:) refers to the kth T Mi hfi)2 T+ hfi MT i 2 i(:;k)
T

cov(')Mi i
T i

Cki i 2)T T i

2
i hiM k hfi+ 2 h'iT m th r th T i
i o)MT i
Mi w
i .
(k;:) th

E
mi
where the notation X(2) = (h'i corresponds to the matrix or vector whose entries are the squares of the
entries of X, Mrefers to the kcolumn of matrix M, and Mrow. *
('TMif)2 q(';f) Mcov(f)Mhfi + h'ih'i + (' h'idiagv(Mcov(f)M); where diagv(X) corresponds to a
vector whose entries are the diagonal entries
Thih'i Mi(:;k) + 2 hfi MT i (cov(') + h'i)M
(m;:)

(m;:)

(:;k)
i

2 = I
of matrix X. 2 = I ; where t he

Xcorrespond T s to the vector Xwith its kentry removed, Mrefers to the matrix formed by removing the kcolumn of M, and Mis the matrix formed by removing its k

(:; k)T k i (m ::)

(co v(f) + hfi hfi

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